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Que 1 :– State and Prove Cauchy's Theorem :–

Theorem :– Suppose G is a finite abelian group and $\frac{P}{O(G)}$ i.e P is a divisor of P(G) where P is a Prime number. Then there is an element $a \neq e \in G$ such that $a^P = e$

Proof :– This theorem is Proved by induction on the order of G

Let us assume that the theorem is true for abelian groups of order less than that of G. we shall Prove that it is also true for G. To Start the induction, we not that the theorem is true for groups of order one.

If G has no proper subgroups then G must be of Prime order because every group of composite order passesses proper subgroups. But P is prime and $P | o(G)$. so $o(G)$ must be equal to P. Also every group of prime order is cyclic.

Therefore each element $a \neq e$ of G will be a generator of G Thus G has P – 1 element are such that $a^n = a^{o(G)} = e$

So now suppose that G has a proper sub group H.

i.e $H \neq \{e\}$ and $H \neq G$

If $\frac{P}{o(H)}$, then by induction hypothesis the theorem is true for H. because.

H is an abelian group and $o(H) < o(G)$ Therefore ∃ an element $b \in H$ $b \neq e$ such that $b^n = e$

So let us suppose that P is not a divisor of $o(H)$. Since G is abelian therefore H is a normal Subgroup of G and So G|H is a quotient group. Since G is abelian

therefore G|H is also abelian

Also we have $o(G|H) = \frac{o(G)}{o(H)} < o(G)$ because $o(H) > 1$

Since $\frac{P}{o(H)}$ and P is not a divisor of $o(H)$. therefore P is a divisor of $o(G)/o(H)$. Hence by our induction hypothesis the theorem is true for the group G|H.

Remebering that H is the identity element of G/H we deduce that $\exists$ an element c in G and $Hc \neq H$ in G/H

Such that $(Hc)^p = H$

Now Hc is not equal to the identity element H of the quotient group G/H and p is a prime. Therefore $(Hc)^p$ implies that in the quotient group G/H

we have $o(Hc) = p$

Also $(Hc)^p = H \Rightarrow Hc^p = H \rightarrow c^p \in H$.

therefore by corollary to Lagranges theorem

we have $(c^p)^{o(H)} = e$

(i.e the identity of H)

$\therefore (c^{o(H)})^p = e$ or $d^p = e$ if we put $d = c^{o(H)}$

Thus d will be the required element of G if we show that $d \neq e$ if possible, let $d = e$.

Then $(Hc)^{o(H)} = Hc^{o(H)} = Hd = He = H$ But in the quotient group G/H, we have $o(Hc) = p$.

Therefore $(Hc)^{o(H)} = H$ (i.e the identity of G/H)

implies that p must be a divisor of $o(H)$ which is a contradiction so,

we cannot have $d = e$

Thus $d \neq e$ and $d^p = e$

Therefore we have completed the induction and this proved the theorem